K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.
Assignment-I
T. Y. B. Sc. (Sem-VI)
MATHEMATICS
MAT-309 (Analysis-III)

Q-1 Define the following terms :

(i) Metric **(ii)** Metric Space **(iii)** Discrete Metric **(iv)** Open Sphere **(v)** Open Set
(vi) Interior point of a set **(vii)** Interior of a set **(viii)** Limit Point of a set
(ix) Closed set. **(x)** Closed Sphere **(xi)** Derived Set **(xii)** Closure of a set
(xiii) Boundary Point of Set **(xiv)** Boundary of a Set.

Q-2 Define discrete metric and prove that it actually a metric on set X

Q-3 Define a metric . If " d " is a metric on X then prove that a function d_1 defined on X as $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ for all $x,y \in X$ is also a metric on X.

Q-4 Prove that every open sphere is an open set in a metric space (X,d).

Q-5 Prove : (i) Arbitrary union of open sets is an open set.
(ii) Finite intersection of open sets is open set in a metric space X.
Does the result hold for any intersection of open sets? Justify.

Q-6 Prove that a subset G of a metric space X is open if and only if G is a union of open spheres in X.

Q-7 Prove that subset A of a metric space X is open in X if and obly if Int(A) = A.

Q-8 Prove that every Closed sphere is a closed set in a metric space (X,d).

Q-9 Prove that a subset F of a metric space X is closed $\Leftrightarrow$ It's complement F' is open in X.

Q-10 Prove that any intersection of closed sets is closed set in a metric space X.
Does the result hold for any union of closed sets? Justify.

Q-11 Answer the following questions in <u>*SHORT*</u> :

(*a*) Give an example of a set which is both open and closed.
(*b*) Give an example to show that the arbitrary intersection of open sets is not open.
(*c*) Define limit point and interior point in a metric space.
(*d*) Define interior and closure of a set in a metric space.
(*e*) Find Int(N) and $\overline{N}$.
(*f*) Find open sphere in usual metric space (R,d).
(*g*) Find all open sets in a discrete metric space .
(*h*) Define boundary point and boundary of a subset A of a metric space X.

– – – × – – – × – – –