K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-I

T. Y. B. Sc. (Sem-VI)

MATHEMATICS

MAT-307 (Abstract Algebra-II)

Q-1 Define the following terms :

(i) Ring **(ii)** Commutative Ring **(iii)** Division Ring **(iv)** A ring with unity element .
(v) Unity element of a Ring **(vi)** Unit element of a Ring. **(vii)** A zero divisor
(viii) An Integral Domain. **(ix)** A Field. **(x)** A Boolean Ring.

Q-2 Show that the set $M_2(Z)$ of all 2×2 real matrices with integral entries forms a ring under usual addition and multiplication of matrices.

Q-3 Show that the set $Z[i] = \{ a + i\,b \,/\, a , b \in Z \}$ forms a commutative ring under usual addition and multiplication of complex numbers.

Q-4 Prove the followings properties in a ring R :

(i) $a\cdot\mathbf{0} = \mathbf{0}\cdot a = \mathbf{0}$ **,** $\forall\, a \in R$, where **0** is the zero element of R.
(ii) $a\cdot(-b) = (-a)\cdot b = -(a\cdot b)$, $\forall\, a, b \in R$.
(iii) $(-1)\cdot a = -a$, for every $a \in R$
(iv) $(-1)\cdot(-1) = 1$ in R.

Q-5 Define a Boolean ring and prove that a Boolean ring is a commutative ring.

Q-6 Define an integral domain and prove that every field is an integral domain. Is the converse true ? Justify your answer in short.

Q-7 Define a Boolean ring and show that $(P(U), \Delta, \cap)$ is a Boolean ring.

Q-8 Define a commutative ring and show that a ring R in which $(a + b)^2 = a^2 + 2ab + b^2$ holds true for all $a, b \in R$ is a commutative ring.

Q-9 Show that $(Z_7, +_7, *_7)$ forms a ring.
Construct operational tables and find multiplicative inverse of all its unit elements.

Q-10 Answer the following questions in *SHORT* :

(*a*) Give an example of a non-commutative finite ring and non-commutative infinite ring.
(*b*) Give an example of a finite field and an infinite field.
(*c*) Give an example of a Boolean ring.
(*d*) Give an example of **(i)** a left ideal which is not a right ideal.
and **(ii)** a right ideal which is not a left ideal.
(*e*) Give an example of a commutative ring which is not a field.
(*f*) Give an example of a division ring which is not a field.
(*g*) Give an example each of a Boolean ring and a non-Boolean ring.
(*h*) Justify whether $(N, +, \cdot)$ forms ring or not.
(*i*) Give an example of an integral domain which is not a field.

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