K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-I

S. Y. B. Sc. (Sem-IV)

MATHEMATICS

MAT-205 (Abstract Algebra-I)

Q-1 Define the following terms :

(i) Relation **(ii)** Reflexive Relation **(iii)**Symmetric Relation **(iv)** Transitive Relation
(v) Equivalence Relation **(vi)** Binary Operation **(vii)** Associative Binary Operation
(viii) Commutative Binary Operation **(ix)** Identity element relative to a binary operation
(x) Group **(xi)** Commutative Group **(xii)** Order of a Finite Group **(xiii)** Order of an element.

Q-2 Define an equivalence relation and determine whether the relation S defined by aSb if $a \neq 4$, $b \neq 4$ on the set Z , is an equivalence relation or not.

Q-3 If (G, $*$) is a group then prove the following properties :

(i) a group (G, $*$) has a unique identity for binary operation $*$.
(ii) every element has a unique inverse in a group (G, $*$).
(iii) For $a, b \in$ G, $(a * b)^{-1} = b^{-1} * a^{-1}$
(iv) For $a, b, c \in$ G, $a * b = a * c \Rightarrow b = c$.

Q-4 Prepare a finite table and show that the set (Z_4 , $+_4$) forms a commutative group .

Q-5 Show that G = $\left\{\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \middle| a, b \in R\right\}$ forms a commutative group under matrix addition.

Q-6 Prove that the set of forth roots of unity is an abelian group under usual multiplication.

Q-7 Prove that the subset G={ $a + b\sqrt{2}$ | a ,b $\in$ Q , $a^2 + b^2 \neq 0$ } of R is a group under usual multiplication of two real numbers.

Q-8 Prove that a group G is commutative if $a^2 = e$, for all $a \in$G.

Q-9 Prove that a Group G is commutative if $(ab)^2 = a^2b^2$, for all $a,b \in$ G.

Q-10 Answer the followings questions in *SHORT* :

(a) Define an equivalence relation and a partition of a set.
(b) Why the set of odd integers is not a group under '+'?
(c) Give an example of non-commutative group.
(d) Give an example of a group of order 4 in which each element is self inverse.
(e) Give an example of a non-associative binary operation on R.

Q-18 Determine whether the following statements are true or false :

(i) The set N of natural numbers forms a group under usual operation of addition.
(ii) The set Z of all integers forms a group under usual operation of addition.
(iii) The set Z of all integers forms a group under usual operation of subtraction.
(iv) The set Q of all rational numbers forms a group under usual operation of multiplication.
(v) The set R of all real numbers forms a group under usual operation of multiplication.

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