Assignment-1

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-IV)

MAT-204 (Advanced Calculus-II)

1. Prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$.
2. Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$.
3. State and prove Duplication formula for beta and gamma functions.
4. Evaluate the integral using beta-gamma functions :
   (i) $\int_0^1 x^5\,(1-x^3)^{10}\,dx$ (ii) $\int_0^\infty \frac{x^4}{(1+x)^{15}}\,dx$
   (iii) $\int_0^\infty x^2\,e^{-x^4}\,dx$ (iv) $\int_0^1 x^6\,(1-x^2)^{\frac{1}{2}}\,dx$
5. Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ and hence evaluate it.
6. Evaluate $\iiint \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ over the region V bounded by planes $x=0, y=0, z=0, x+y+z=1$.
7. Evaluate $\iint_E (x^2-y^2)\,dx\,dy$, where $E = \{(x,y) | 0 \le x \le 1, y \ge 1, y \le x+1\}$.
8. Evaluate $\int_0^1 \int_0^\pi \int_0^\pi y \sin z\,dx\,dy\,dz$.

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