

Assignment-2

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-V)

MAT-305 [Elective Paper]
(Discrete Mathematics)

- 1 Prove that every chain is a distributive lattice.
- 2 Prove that the direct product of any two distributive lattices is a distributive lattice.
- 3 Let $\langle L, *, \oplus \rangle$ be a distributive lattice, for $a, b, c \in L$, prove that
$$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a).$$
- 4 State and prove De' Morgan's laws in a Boolean algebra.
- 5 Show that in a Boolean algebra
$$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'.$$
- 6 Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean algebra and $S \subseteq B$. If S is closed under $\{*, '\}$ or $\{\oplus, '\}$, then show that S is a sub-Boolean algebra of B .
- 7 Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean algebra, A be the set of all atoms of B and $x_1, x_2 \in B$. Prove that
 - 1) $A(0) = \phi$
 - 2) $A(1) = A$
 - 3) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$
 - 4) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$
 - 5) $A(x') = A - A(x)$
- 8 State and prove Stone's representation theorem.
- 9 Prove that the product of two distinct minterms is zero.
- 10 Prove that the sum of all minterms in n variables is 1.
- 11 Find POS and SOP canonical forms of the following Boolean expression (**Any one**)
 - 1) $x_1 * x_2$
 - 2) $x_1 \oplus x_2$
 - 3) $(x_1 \oplus x_2)' * x_3$
 - 4) $(x_1 * x_2) \oplus x_3$
 - 5) $(x_1 \oplus x_2)'$
 - 6) $x_1 \oplus (x_2 * x_3)'$
- 12 Define Boolean algebra.
- 13 Define sub-Boolean algebra.
- 14 Define direct product of two Boolean algebras.
- 15 Define atom in a Boolean algebra.
- 16 Define Boolean homomorphism.
- 17 Define Boolean isomorphism.
- 18 Define Boolean expression.
- 19 Define SOP canonical form.
- 20 Define POS canonical form.

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