Assignment-2

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-V)

MAT-303 (Complex Variables and Fourier Series)

1. Find the image of the region R in z-plane, bounded by the lines $x = 1, x = 2, y = 0, y = 1$, under the mapping $w = z + (2 - i)$.

2. Find the image of the infinite strips (a) $\frac{1}{4} < y < \frac{1}{2}$ and (b) $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Show them graphically.

3. If $w = f(z)$ is a bilinear transformation which associate z_1, z_2, z_3 and z_4 to w_1, w_2, w_3 and w_4 respectively then prove that $\frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)} = \frac{(w_1-w_2)(w_3-w_4)}{(w_1-w_4)(w_3-w_2)}$.

4. Find the bilinear transformation which maps $-1, 0, 1$ of z-plane onto the points $-i, 1, i$ of w-plane respectively.

5. Find a bilinear transformation that maps the points $\infty, -i, 0$ of z-plane in to $0, 1, \infty$ of w-plane respectively.

6. Find bilinear transformation which maps $i, 1, \infty$ of z-plane to $1, i, 0$ of w-plane respectively.

7. Obtain the image of $|z - i| < 3$ under the transformation $w = \frac{iz+1}{2i+z}$.

8. Prove that if $f(z)$ is analytic at z_0 and $f'(z_0) \neq 0$, then $w = f(z)$ is conformal at z_0.

9. Prove that the magnitude and the direction of angle between the lines $y = 2x$ and $y = x - 1$ remains same under the mapping $w = f(z) = z^2$.

10. Prove that the magnitude and the direction of angle between the lines $y = 2x$ and $y = x - 1$ remains same under the mapping $w = f(z) = \frac{1}{z}$.

11. If the series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ converges uniformly to f in $[-\pi, \pi]$, then prove that it is the Fourier series for f on $[-\pi, \pi]$.

12. State and prove Euler's formula for the Fourier series.

13. State and prove Bessel's inequality for the Fourier series.

14. Find the Fourier series for the function $f(x) = x^2$ in $[-\pi, \pi]$ and deduce that (i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$, (ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

15. Find the Fourier series for the function $f(x) = x + x^2$ in $[-\pi, \pi]$.

16. Obtain Fourier series for the function $f(x) = x^2 - x$ in $[0, \pi]$.

17. Obtain Fourier series for the function $f(x) = e^{ax} \sin bx$ in $[0, \pi]$.

18. Express the function $f(x) = x + x^2$ as a Fourier series in $[-\pi, \pi]$.

19. Find the half range sine series of $f(x) = \frac{\pi}{4}\cos x$ in $(0, \pi)$.

20. Find a sine series for the function $f(x) = x$ for $0 < x < \frac{\pi}{2}$ and $f(x) = 0$ for $\frac{\pi}{2} < x < \pi$.

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