

Assignment-2

KKSJ MANINAGAR SCIENCE COLLEGE
B. Sc. (Sem-V)
MAT-302 (Analysis-I)

1. Using definition verify that $\lim_{x \rightarrow 6} x^2 + 2x - 7 = 41$, also find δ corresponding to $\epsilon = 0.1$.
2. Prove that if limit of a function exists then it is unique.
3. Prove in usual notations that $\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$.
4. Prove that a real valued function f is continuous at a if and only if for every sequence (x_n) in domain of f with $x_n \rightarrow a \Rightarrow f(x_n) \rightarrow f(a)$.
5. If function f is continuous at a and function g is continuous at $f(a)$ then function $g \circ f$ is continuous at a .
6. Let $f(x) = \frac{(x-1)|x-2|}{x^2-3x+2}$, discuss the continuity at $x = 1$ and $x = 2$.
7. Show that $f(x) = x^2 + 3x + 6$ is uniform continuous on $[0, 9]$.
8. Show that $f(x) = x^2 + 2x + 3$ is uniform continuous on $[0, 4]$.
9. Show that $f(x) = \frac{1}{x}$ is not uniform continuous on $(0, 1)$.
10. State and prove Intermediate Value Theorem (IVT).
11. Let f be strictly increasing on $I \subset \mathbb{R}$; then prove that f^{-1} exists and is also strictly increasing on $f(I)$.
12. State and prove Extreme Value Theorem (EVT).
13. Define differentiable function. Prove that every differentiable function is continuous function. Does converse hold? Justify your answer.
14. State and prove Rolle's Theorem.
15. Verify Rolle's Mean Value theorem for the function $f(x) = e^x(\cos x - \sin x)$ in the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$ and find c such that $f'(c) = 0$.
16. State and prove Mean Value Theorem.
17. Using MVT prove that $\frac{y-x}{1+y^2} < \tan^{-1} y - \tan^{-1} x < \frac{y-x}{1+x^2}$; $0 < x < y$.
18. State and prove generalized (extended) Lagrange mean value theorem.
19. State and prove second L' Hospital's rule.
20. Evaluate $\lim_{x \rightarrow 0} \left(\frac{2^x + 3^x + 5^x}{3} \right)^{\frac{1}{x}}$.
