K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-II

T. Y. B. Sc. (Sem-V)

MATHEMATICS

MAT-301 (Linear Algebra-II)

Q-1 Define orthogonal vectors, orthogonal set and orthonormal set in an inner product space.

Q-2 Define projection of a vector *u* along nonzero vector *v* of an an inner product space V.

Q-3 Prove that a finite dimensional inner product space has an orthogonal basis.

Q-4 Prove that a finite dimensional inner product space has an orthonormal basis.

Q-5 Apply the Gram-Schmidt orthogonalization process to the basis
B = {(–1,1,0), (0, –1,1), (1,0, –1) } in order to get orthogonal basis for V_3 .

Q-6 Find an orthogonal basis for R^3 by applying the Gram-Schmidt orthogonalization process to the basis B = { (2,1,1), (1,2,1), (1,1, 2) } of R^3.

Q-7 Find an orthonormal basis for V_3 by applying the Gram-Schmidt orthogonalization process to the basis B = { (0,1,1), (1,0,1), (1,1, 0) } of V_3.

Q-8 Apply the Gram-Schmidt orthogonalization process on the set
A = {(0, 1,1,1), (1,1,0,1), (1,1, 1,0) } in order to get orthogonal set in V_4.

Q-9 Prove that any orthogonal set of nonzero vectors is linearly independent.

Q-10 In usual notations prove the followings :

(i) If *x* and *y* are orthogonal vectors of an inner product space V then prove that
$\|x + y\|^2 = \|x\|^2 + \|y\|^2$.

(i) If x_i's for $i = 1, 2,\ldots$, n are orthogonal vectors of an inner product space V then prove that $\|\sum_{i=1}^{n} x\mathrm{i}\|^2 = \sum_{i=1}^{n} \|x\mathrm{i}\|^2$.

Q-11 State and prove the Cauchy-Schwarz inequality.

Q-12 State and prove the triangle inequality.

Q-13 Define an orthogonal linear map.
If (V, < , >) is an inner product space then prove that a linear map T : V $\rightarrow$V is an orthogonal linear map **if and only if** $\|T(x)\| = \|x\|$ for all $x \in V$.

Q-14 Explain the role of a 2×2 determinant in the area of a parallelogram.

Q-15 If V is a real vector space of dimension *n* and if *det* : $V^n \rightarrow R$ is a function satisfying the expected properties of the determinant then prove the followings :

(i) $det\,(v_1 , v_2 , \ldots , v_n) = 0$ if $v_i = \bar{0}$ for some $i = 1,2,\ldots$, n.

(ii) $det\,(v_1 , \ldots,v_i ,\ldots, v_j,\ldots,v_n) = det\,(v_1 ,\ldots, v_i + \alpha v_j ,\ldots,v_j,\ldots, v_n)$, for $i \neq j$ and $\alpha \in R$.

(iii) $det\,(v_1 , v_2 , \ldots , v_n) = 0$ if $\{v_1 , v_2 , \ldots , v_n \}$ is Linearly Dependent.

(iv) $det\,(v_1 , v_2 , \ldots, x_j + y_j,\ldots , v_n) = det\,(v_1 , v_2 , \ldots, x_j,\ldots , v_n) + det\,(v_1 , v_2 , \ldots, y_j,\ldots , v_n)$

(v) $det\,(v_1 , \ldots , v_i ,\ldots, v_j ,\ldots , v_n) = - det\,(v_1 , \ldots , v_j ,\ldots, v_i ,\ldots , v_n)$ for $i \neq j$.

Q-16 Define an *r*-linear map.
Also show that if V is a real vector space of dimension *n* and then the determinant mapping $det : V^n \to R$ is an *n*-linear map

Q-17 If (V, <,>) is an inner product space then show that the map $f : V \times V \to R$ defined as $f(x, y) = < x, y >$ is a bilinear map (i.e. 2-linear map).

Q-18 Apply the properties of determinant to find *det*A if A = $\begin{bmatrix} a & b & c \\ u & v & w \\ x & y & z \end{bmatrix}$.

Q-19 If A = $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & -1 \\ 2 & 0 & 4 & 2 \\ 7 & 3 & 1 & -1 \end{pmatrix}$ then compute *det*A without expansion.

Q-20 If A = $\begin{pmatrix} 1 & 5 & 0 & 0 \\ 2 & 0 & 8 & 0 \\ 3 & 6 & 9 & 0 \\ 4 & 7 & 10 & 1 \end{pmatrix}$ then compute *det*A by using properties of the determinant map.

Q-21 State and Prove the Cramer's rule for solving a system of linear equations.

Q-22 Apply the Cramer's rule to solve the following system of linear equations :

$$x + y = 2$$
$$y + z = 8$$
$$z + x = 4.$$

Q-23 Apply the Laplace Expansion about the last row of A = $\begin{pmatrix} 1 & 0 & 3 & 4 \\ 0 & -1 & 4 & 5 \\ 1 & 2 & 0 & 3 \\ 1 & 0 & 1 & -1 \end{pmatrix}$ to find *det*A .

Q-24 Define eigen value and eigen vector of a linear operator T : V→V.
Also find eigen value and eigen vector of the linear map $T : V_2 \to V_2$ defined as

$$T(x_1, x_2) = (x_2, x_1)$$, if exists.

Q-25 Express the characteristic equation of 2×2 matrix A in terms of Trace of A and *det* A.
Also Prove that a 2×2 real and symmetric matrix has only real eigen values.

Q-26 State and prove the Cayley-Hamilton's Theorem.

Q-27 If A = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then verify the Cayley-Hamilton's theorem for A also A^{-1} if exists.

Q-28 Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, also find the modal matrix which diagonalizes A.

Q-29 Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

Q-30 Find eigen values and eigen vector corresponding to any one eigen value of $A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix}$

Q-31 Define a quadric.

Q-32 Identify the quadric in R^3 given by

$$f(x, y, z) \equiv 4xz + 4y^2 + 8y + 8 = 0$$

Q-32 Answer the following questions in short :

(*d*) Define orthonormal set and give one example of it.

(*e*) Define : (i) eigen basis (ii) Diagonalizable matrix

(*f*) State the Cramer's rule for solving a system of linear equations.

(*g*) State the Laplace Expansion.

(h) Define eigen value and eigen vector of an endomorphism.

(i) Define a bilinear map and a quadric .

(*e*) Define orthonormal set and give one example of it.

(*f*) State the Laplace Expansion.

(g) Find *det*A if $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

(h) Define a bilinear map and a quadric .

(i) Write any equation of a hyperboloid of two sheets.

(*d*) State the Cauchy-Schwarz inequality.

(*f*) Define eigen value and eigen vector of linear operator.

(*g*) State the Cramer's rule for solving a system of linear equations.

(*d*) State the triangle inequality in terms of the norm of vectors.

(*e*) Define an inner product and an inner product space.

(*f*) State the Cramer's rule for solving a system of linear equations.

(*g*) State the spectral theorem.

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