K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-II

S. Y. B. Sc. (Sem-III)

MATHEMATICS

MAT-202 (Linear Algebra-I)

Q-1 Define a Linear Combination, Trivial Linear Combination and the Span of a set.

Q-2 Define a **Linearly Dependent** subset and a **Linearly Independent** subset of a vector space V.

Q-3 Prove that the set $\{ v \}$ is Linearly Dependent (LD) ***iff*** $v = \bar{0}$.

Q-4 Prove that the every subset of a vector space V containing the zero vector is linearly dependent in V

Q-5 In a Vector Space V, if $\{ v_1 , v_2 ,\ldots, v_n \} \subset V$ and $v \in [v_1 , v_2 ,\ldots, v_n]$ then prove that $\{ v, v_1, v_2 ,\ldots, v_n \}$ is linearly dependent in V.

Q-6 In a Vector Space V, if $\{ v_1 , v_2 ,\ldots, v_n \}$ is linearly independent and $v \notin [v_1 , v_2 ,\ldots, v_n]$ then prove that $\{ v , v_1, v_2 ,\ldots, v_n \}$ is linearly independent in V.

Q-7 Prove that every superset of a linearly dependent set is linearly dependent in a vector space V.

Q-8 Prove that every subset of a linearly independent set is linearly independent in a vector space V.

Q-9 If $A = \{ v_1, v_2,\ldots, v_n\}$,where $v_1 \neq \bar{0}$ and if one of the vectors v_i of A, for some $i = 2,3,..n$ is a linear combination of $v_1, v_2,\ldots, v_{i-1}$ then prove that A must be linearly dependent in V.

Q-10 Define a Basis of a vector space .

Q-11 If $A = \{ u_1, u_2,\ldots, u_m\}$ is LI subset of a vector space V and if $B = \{ v_1, v_2,\ldots, v_n\}$ spans the vector space V then prove that $m \leq n$.

Q-12 Prove that any two bases of a finite dimensional vector space has equal number of elements.

Q-13 Show that $B = \{ (-1,1,1) ,(1, -1,1) , (1,1, -1)\}$ is a basis for the vector space V_3.

Q-14 If $W = \{ (x_1+ x_2, x_1- x_2, 0) / x_1, x_2 \in R \}$ then find a basis of W and hence find *dim*W.

Q-15 Define a Linear Mapping.

Q-16 If $T : U \to V$ is linear then prove that

$$T(\alpha_1 u_1 + \alpha_2 u_2 + \ldots + \alpha_n u_n) = \alpha_1 T(u_1)+ \alpha_2 T(u_2) + \ldots + \alpha_n T(u_n),$$

where α_i's are scalars and u_i's are vectors of U for each $i = 1,2,\ldots,n$.

Q-17 Determine which of the following mappings are linear :

(i) $P : V_3 \to V_3$ defined as $T(x,y,z) = (y, z, x), \ \forall (x,y,z) \in V_3$

(ii) $R : V_2 \to V_3$ defined as $S(x,y) = (x, xy, y), \ \forall (x,y) \in V_2$

(iii) $S : V_3 \to V_4$ defined as $T(x,y,z) = (x, y, z, z+1), \ \forall (x,y,z) \in V_3$

(iv) $T : V_3 \to V_3$ defined as $T(x,y,z) = (x-y, y-z, z-x), \ \forall (x,y,z) \in V_3$

Q-18 Define the kernel N(T) of a linear map $T: U \to V$ and prove that N(T) is a subspace of U.

Q-19 Define the range R(T) of a linear map $T: U \to V$ and prove that R(T) is a subspace of V.

Q-20 In usual notations prove that a linear map T: U$\rightarrow$V is one-one ***iff*** N(T) $=\{\bar{0}_U\}$.

Q-21 Define the Rank and the Nullity of a linear mapping.

Q-22 Find the range and kernel of the linear map

S : $V_3 \rightarrow V_2$ defined as $T(x,y,z) = (x+y, y+z)$, $\forall (x,y,z) \in V_3$.

Q-23 Find the range and kernel of the linear map

T : $V_3 \rightarrow V_3$ defined as $T(x,y,z) = (x, y+z, z)$, $\forall (x,y,z) \in V_3$.

Q-24 If T : $V_3 \rightarrow V_3$ is the linear map defined by

T(1,0,0)= (1,0,0) , T(0,1,0)= (1,1,0) , T(0,0,1)= (1,1,1)

Then find R(T) and N(T).

Q-25 State and Prove the Rank-Nullity Theorem.

Q-26 Verify the Rank_Nullity Theorem for the following Linear Mappings :

(i) R : $V_2 \rightarrow V_3$ defined as $T(x,y) = (x, x+y, y)$, $\forall (x,y) \in V_2$

(ii) S : $V_3 \rightarrow V_2$ defined as $T(x,y,z) = (x-y, y-z)$, $\forall (x,y,z) \in V_3$

(iii) T : $V_3 \rightarrow V_3$ defined as $T(x,y,z) = (x, x+y, x+z)$, $\forall (x,y,z) \in V_3$.

Q-27 Define addition and scalar multiple of linear mappings.

Q-28 If U and V are vector spaces defined over the same field then define the space L(U,V).

Q-29 Explain the concept of the matrix associated with a linear map.

Q-30 Explain the concept of the linear map associated with a matrix.

Q-31 Find the matrix (T: B_1, B_2) if T : $V_3 \rightarrow V_2$ is defined as

$T(x,y,z) = (x-y, y-z)$, $\forall (x,y,z) \in V_3$,

B_1= { (0,1,1) ,(1, 0,1) , (1,1, 0)} and B_2= { (1,0) ,(0,1) }

Q-32 Find the linear map T : $V_3 \rightarrow V_2$ so that the matrix A= (T: B_1, B_2) if

$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$, B_1= { (1,1,1) ,(1, 2,3) , (1,0, 0)} and B_2= { (1,1) ,(1,–1) }

Q-33 If $\mu_{m,n}$ is the vector space of all $m \times n$ real matrices then in usual notations prove that

$dim\ \mu_{m,n} = m \times n$

Q-34 Answer the followings in short :

(i) Define the Dimension of a finite dimensional vector space.

(ii) Define a linear transformation.

(iii) Explain Why A = {(1,0) , (0,1) , (1,1) } is not basis for V_2.

(iv) Explain Why B = {(1,0) , (2,0) } is not basis for V_2.

(v) Determine whether C = {(1,0) , (0,0) } is LI or LD .

(vi) Define an Isomorphism.

(vii) What is L(U ,V) ? Explain in short .

(viii) Define the Kernel of a linear map T: U$\rightarrow$V.

(ix) State the Rank Nullity Theorem

(x) State any two properties of a linear transformation .

– – – × – – – × – – –