Assignment-2

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-III)

MAT-201 (Advanced Calculus-I)

1. State and prove Euler's theorem for a function of two variables.
2. If $u = \phi(H)$ is a function of a homogeneous function $H = f(x, y)$ of degree m, whose partial derivatives of second order exist then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = m\frac{f(u)}{f'(u)} = G(u)$ (say), and $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = G(u)[G'(u) - 1]$.
3. Let a real valued function f, defined on an open domain $E \subset R^2$, and differentiable at $(a, b) \in E$. Prove that the necessary condition that f has an extreme value at (a, b) are $f_x(a, b) = 0$, $f_y(a, b) = 0$.
4. If $u = \text{cosec}^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt[3]{x}+\sqrt[3]{y}}\right)^{\frac{1}{2}}$, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \frac{1}{144}\tan u(13 + \tan^2 u)$.
5. If $u = \tan^{-1}\frac{x^3+y^3}{x+y}, x + y \neq 0$, prove that $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \sin 4u - \sin 2u$.
6. Find the extreme values of $f(x, y) = 2(x - y)^2 - x^4 - y^4$.
7. Find the extreme values of $f(x, y) = x^3y^2(1 - x - y)$.
8. State and prove Tyalor's theorem with remainder.
9. Find first two non-zero terms of the expansion of $f(x, y) = \sin x \sin y$ in the powers of x and y.
10. Expand $f(x, y) = x^3 + 2x^2y + 3y^2 - 5xy + 3y$ in the powers of $x - 1$ and $y + 2$.
11. Find the double points of the curve $f(x, y) = x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.
12. Find the double points of the curve $f(x, y) = x^4 + y^3 - 2x^3 + 3y^2 = 0$.
13. Find the equation of radius of curvature of a Cartesian curve.
14. Find the equation of radius of curvature of a Polar curve.
15. Find the radius of curvature for the cubic $y = 2x^3 - x + 3$ at the point $x = 1$.
16. Find the curvature and radius of curvature of the parabola $y = x^2$ at the origin.
17. Find the radius of curvature for $r = a(1 - \cos\theta)$.
18. Find the radius of curvature for $\frac{2a}{r} = 1 + \cos\theta$.

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