K. K. Shah Jarodwala Maninagar Science College, Rambaug, Ahmedabad – 380015.

Statistics Department

Assignment – No. – 01 **Statistics paper – STA - 201**

1 Define the terms: random variable with its types, probability mass function, Probability distribution Function.

2 State and prove the properties of the distribution function of a random variable X.

3 For a random variable X, p.m.f is given as under:

$$P(x) = c\,(x(x+1)) \qquad , x = -2, -1, 0, 1, 2$$
$$= 0 \qquad otherwise$$

Where c is constant to be determined.

Then, find $P(x > 1), P(-1 \le X \le 1), P(X < 1)$. Also find distribution function.

4 If the probability density function of a random variable is

$$f(x) = \begin{cases} k\, e^{-2x} & , x > 0 \\ 0 & , otherwise \end{cases}$$

Determine the value of k. Also, find the distribution function.
Evaluate P (2 < X < 5), P (X ≤ 4).

5 Define probability density function.

For the continuous distribution of a random variable X with the probability density function

$$f(x) = \begin{cases} x & 0 < X \le 1 \\ \dfrac{3-x}{4} & 1 < x \le 4 \\ 0 & elsewhere \end{cases}$$

Find the distribution function and calculate F(2)

6 Define Mathematical Expectation.In usual notation prove that

E (X + Y) = E (X) + E (Y)

7 Define moments, Central moments and moment generating function

8 State and prove the properties of moment generating function

9 A random variable X represents the number of heads in a toss of a coin. The coin is weighted so that P (H) = 2 / 3, and P (T) = 1 / 3. The coin is tossed 3 times.

Find E(X), E (2X+3), V(X), V (5X – 4).

10 If $f(x) = a + bx^2, 0 < x < 1$, is a probability density function of a random variable X, with mean $\frac{2}{3}$.

Then, find a and b.

11 In usual notations, prove E(X * Y) = E(X) * E(Y)

12 For a random variable X, p.d.f is given as under:

$$P(x) = c\,(x(x-1)) \qquad ,0 \le x \le 2$$
$$= 0 \qquad otherwise$$

Where c is constant to be determined.

Then, find $P(x > 0), P(-1 \le X \le 0), P(X \le 2)$. Also find distribution function.

13 The length of (in hours) X of a certain type of electronic chip is supposed to be a continuous random variable with a p.d.f as

$$f(x) = \frac{a}{x^2} \;;\; 20 < X < 30$$
$$= 0 \qquad ; otherwise$$

Where a is a constant to be determined .Then find the probability $P(22 < X < 28)$

14 In usual notation prove that E (X Y) = E (X) E (Y)

Given the following table

X	1	2	3	4	5	6	7
P(x)	3k	15k	20k	24k	20k	15k	3k

Where k is a constant to be obtained. Compute E(X), E (2X-1), E(X +2), V (3X+1)

15 If a r.v X has a probability distribution as shown in the following table:

X	0	1	2	3	4	5	6	7
P(x)	0	$a+7a^2$	$2a^2$	a^2	2a	2a	2a	a

Determine (i) a, (ii) $P(x < 5), P(x \ge 4), P(2 < x \le 6)$

16 A random variable X follows a probability density function $f(x) = \begin{cases} kx(2-x) & ,0 \le x \le 2 \\ 0 & ,elsewhere \end{cases}$

Then find *k*, variance, β_2.

17 In usual notations, prove $\mu_4 = k_4 + 3k_2^2$

18 Define factorial moments.

19 In usual notations, prove that

E(CX) = C E(X) , where C is a constant.

E(AX+B) = A E(X) + B

20 In context to mathematical expectation, Prove that, if random variables X and Y are independent,

$V(AX+BY) = A^2\, V(X) + B^2\, V(Y)$

21 In context to mathematical expectation, Prove that, for random variables X and Y,

$V(AX+BY) = A^2\, V(X) + B^2\, V(Y) + 2AB\, Cov(X,Y)$

22 A random variable X follows a probability density function $f(x) = \begin{cases} kx(1-x) & ,0 \le x \le 1 \\ 0 & ,elsewhere \end{cases}$

Then, show that *mean = median = mode = 1/2*.