

Assignment-1

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-V)

MAT-305 [Elective Paper]

(Discrete Mathematics)

Date: 31-07-2017

- 1 Let $X = \{a, b, c, d\}$. Which of the following relations are partial order relations on?
 - (1) $R_1 = \{\langle a, a \rangle, \langle b, b \rangle\}$
 - (2) $R_2 = \{\langle a, a \rangle, \langle b, b \rangle, \langle a, b \rangle, \langle b, a \rangle\}$
 - (3) $R_3 = \{\langle a, a \rangle, \langle a, b \rangle, \langle b, a \rangle\}$
 - (4) $R_4 = \{\langle a, b \rangle, \langle b, a \rangle\}$
 - (5) $R_5 = \{\langle a, a \rangle, \langle b, b \rangle, \langle c, c \rangle, \langle d, d \rangle\}$
- 2 Show that $\langle N, D \rangle$ is a poset but not a chain.
- 3 Show that $\langle P(X), \subseteq \rangle$ is a poset.
- 4 Let $\langle P, \leq \rangle$ be a poset and $A \subset P, A \neq \phi$. Show that $\langle A, \leq \rangle$ is a poset.
- 5 Explain Hasse diagram and draw the Hasse diagram of $\langle S_{30}, D \rangle$.
- 6 Show that $\langle P(X), \subseteq \rangle$ is a lattice.
- 7 Prove that every chain is a lattice. Is the converse true? Justify.
- 8 Show that $\langle N, D \rangle$ is a lattice.
- 9 For a lattice $\langle L, \leq \rangle$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.
- 10 Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L$, prove that $b \leq c \Rightarrow \begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$

Or
State and prove isotonicity property in a lattice.
- 11 Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L$, prove that $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

Or
State and prove distributive inequalities in a lattice.
- 12 Let $\langle L, \leq \rangle$ be a lattice. For any $a, b, c \in L$, prove that $a \leq c \Rightarrow a \oplus (b * c) \leq (a \oplus b) * c$.

Or
State and prove modular inequality in a lattice.
- 13 For a lattice $\langle L, \leq \rangle$ prove that (i) $a * b = b * a$ (ii) $a * (b * c) = (a * b) * c$ (iii) $a * (a \oplus b) = a$.
- 14 Prove that the direct product of two lattices is a lattice.
- 15 Show that $\langle S_{30}, *, \oplus \rangle$ i.e. $\langle S_{30}, D \rangle$ and $\langle P(X), \cap, \cup \rangle$ lattices are isomorphic, where $X = \{a, b, c\}$.
- 16 Prove that every chain is a distributive lattice.

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