K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-I

T. Y. B. Sc. (Sem-V)

MATHEMATICS

MAT-304 (Mathematical Programming)

Q-1 Define the following terms :

(*a*) Define a Euclidean Space.

(*b*) Define a convex linear combination, convex set and a vertex of a convex set.

(*c*) Define a slack variable and surplus variable.

(*d*) Define a solution of an LP Problem.

(*e*) Define a basic solution of an LP Problem.

(*f*) Define a basic feasible solution of an LP Problem.

(*g*) Define a degenerate basic solution a non-degenerate basic solution of LPP.

(*h*) Define an artificial variable.

(*i*) Define an optimum and an unbounded solution of an LP Problem.

(*j*) Define the Dual of a Linear Programming Problem

Q-2 Prove that $K \subset R^n$ is a convex set if and only if every convex linear combination of elements in K also belongs to K.

Q-3 Prove that the intersection of two convex sets is a convex set.

Q-4 Define a Convex Hull of a set also prove that a convex hull of a set is always a convex set.

Q-5 Show that a set of all convex linear combinations of points in a set is a convex set.

Q-6 If $S \subset E_n$ then prove that the convex hull of S, namely [S] is the set of all convex linear combinations of points in S.

Q-7 Define a convex Polyhedron and prove that a convex polyhedron is a convex set.

Q-8 Determine convexity of the sets : $S_1 = \{x \in E^n / \|x\| \leq 1\}$ and $S_2 = \{x \in E^n / \|x\| \geq 4\}$.

Q-9 If $S = \{ x \in E^n / \| x \| = 1 \}$ then explain whether S is convex or not.

Q-10 If S_F is a nonempty set of all feasible solutions of a LP Problem then prove that S_F is a convex set.

Q-11 A manufacturer of furniture makes two products: chairs and tables. These products are processed on two machines A and B. A chair requires 2 hours of processing time on machine A and 3 hours on machine B. A table requires 5 hours of processing time on machine A and no time on machine B. There are 16 hours of time available for machine A and 20 hours on machine B during any working day.

Profit gained by the manufacturer from a chair is Rs 50 and that of a table is Rs 90.What should be the daily production of each of the t products?

Formulate the linear programming problem.

Q-12 A firm manufactures two types of products A and B and sells them at a profit of Rs 20 and Rs 30 respectively.Each product is processed on two machines G and H. Type A requires 2 minute of processing time on G and 3 minutes on H whereas

type B requires 3 minutes of processing time on G and 2 minute on H.The machine G is available for not more than 6 hours 40 minutes while machine H is available for 3 hours 20 minutes during any working day.

How many product of each type should the firm produce in order to get maximum profit?

Formulate the linear programming problem.

Q-13 A manufacturer produces two types of models M_1 and M_2. Each M_1 model requires 4 hours of grinding and 5 hours of polishing. Each M_2 model requires 5 hours of grinding and 3 hours of polishing. The manufacturer has two grinders and 3 polishers .Each grinder works 40 hours a week and each polisher works for 60 hours a week.
Profit on an M_1 model is Rs.12 & on an M_2 model is Rs 15.
Whatever is produced in a week is sold in the market .

How should the manufacturer allocate his production capacity to the two types of models so that he may make a maximum profit in a week ?

Formulate the LP problem .

Q-14 A person requires 10,12 and 12 units of chemicals A,B and C respectively for his garden. A liquid product contains 5,2 and 1 units of A,B and C respectively per jar. A dry product contains 1,2 and 4 units of A,B and C respectively per carton. If the liquid product sells for Rs 10 per jar and the dry product sells for Rs 12 per carton, how many of each should be purchased in order to minimize the cost and meet the requirements ?

Formulate the linear programming problem.

Q-15 Solve the following LP Problem by Simplex Method :

Minimize $Z = 12x_1 + 5x_2$

Subject to $5x_1 + x_2 \leq 15$

$7x_1 + 2x_2 \leq 14$ and $x_1, x_2 \geq 0$.

Q-16 Solve the following LP Problem by Simplex Method :

Maximize $Z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$ and $x_1, x_2 \geq 0$.

Q-17 Use the Simplex Method to solve the following LP Problem :

Maximize $Z = 107x_1 + x_2 + 2x_3$

Subject to $14x_1 + x_2 - 6x_3 + 3x_4 = 7$

$16x_1 + \frac{1}{2}x_2 - 6x_3 \leq 5$

$32x_1 + x_2 - 12x_3 \leq 10$

$3x_1 - x_2 - x_3 \leq 0$ and $x_1, x_2, x_3, x_4 \geq 0$.

Q-18 Solve the following LP Problem by Simplex Method :

Maximize $Z = x_1+x_2+3x_3$

Subject to $3x_1+2x_2+x_3 \leq 3$

$2x_1+x_2+2x_3 \leq 2$ and $x_1, x_2, x_3 \geq 0$.

Q-19 Use the Simplex Method to solve the following LP Problem :

Maximize $Z = 4x_1 + x_2 + 3x_3 + 5x_4$

Subject to $4x_1 - 6x_2 - 5x_3 + 4x_4 \geq -20$

$3x_1 - 2x_2 + 4x_3 + x_4 \leq 10$

$8x_1 - 3x_2 + 3x_3 + 2x_4 \leq 20$ and $x_1, x_2, x_3, x_4 \geq 0$.

Q-20 Solve the following LPP by Simplex Method :

Maximize $Z = 3x_1+ 5x_2$

Subject to $3x_1+ 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$ and $x_1, x_2 \geq 0$.

Q-21 Solve the following LPP by Simplex Method :

Maximize $Z = 4x_1+ 5x_2$

Subject to $x_1+x_2 \leq 3$

$5x_1+2x_2 \leq 8$

$3x_1+8x_2 \leq 10$ and $x_1, x_2, x_3 \geq 0$.

Q-22 Solve the following LPP by Simplex Method :

Maximize $Z = x_1+x_2+3x_3$

Subject to $3x_1+2x_2+x_3 \leq 3$

$2x_1+x_2+2x_3 \leq 2$ and $x_1, x_2, x_3 \geq 0$.

Q-23 Find the dual of the following LP Problem and verify that the dual of dual is primal :

Maximize $Z = 4x_1+x_2+3x_3$

Subject to $4x_1+ 6x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2+4x_3 \leq 20$

$8x_1 + 3x_2+3x_3 \leq 30$ and $x_1, x_2, x_3 \geq 0$

Q-24 Find the dual of the following LP Problem and verify that the dual of dual is primal :

Minimize $Z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \geq 2$

$x_1 + x_2 \geq 3$ and $x_1, x_2 \geq 0$.

Q-25 Determine whether the following statements are true or false :

(i) Every Linear Programming Problem has a solution.

(ii) Some Linear Programming Problem has an unbounded solution.

(iii) An empty set is a convex set.

(iv) Some LPP cannot be solved by Graphical Method.

(v) Every Linear Programming Problem can be solved by usual simplex method.

(vi) Every Linear Programming Problem has a dual LPP.

– – – × – – – × – – –