Assignment-1

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-V)

MAT – 303 (Complex Variables and Fourier Series)

Date: 31-07-2017

1 State and prove the triangular inequality for the complex numbers, hence show that $\big||z_1| - |z_2|\big| \le |z_1 - z_2|$; for $z_1, z_2 \in C$. Prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}, z_2 \ne 0$, for $z_1, z_2 \in C$.

2 Define argument of a complex number $z = x + iy$.
If z_1 and z_2 are complex numbers such that $|z_1 + z_2| = |z_1 - z_2|$ then prove that $\arg z_1 - \arg z_2 = \frac{\pi}{2}$.

3 Prove that z is either real or pure imaginary if and only if $(\bar{z})^2 = z^2$.

4 State and prove De Moivre's theorem.
OR
Prove that for any rational number n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

5 Find all values of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$.

6 State at least ten algebraic properties of the complex numbers.
Show that $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$.
Also find the four roots of the equation $z^4 + 4 = 0; z \in C$.

7 Derive four forth roots of 1.

8 If $\sin(\alpha + i\beta) = x + iy$, then prove that

(i) $\frac{x^2}{\cosh^2\beta} + \frac{y^2}{\cosh^2\beta} = 1$

(ii) $\frac{x^2}{\sin^2\alpha} - \frac{y^2}{\cos^2\alpha} = 1$.

9 Define trigonometric and hyperbolic functions for the complex variables. Show that $|\sin z|^2 + |\cos z|^2 = \cosh 2y = ch\,2y; z \in C$. Also, express $\sqrt{3} - i$ in the exponential form.

10 Suppose that $z_n = x_n + iy_n (n = 1, 2, 3, \ldots)$ and $z = x + iy$. Then prove that $\lim\limits_{n \to \infty} z_n = z \Leftrightarrow \lim\limits_{n \to \infty} x_n = x$ and $\lim\limits_{n \to \infty} y_n = y$.

11 Is there any relation between the trigonometric and hyperbolic complex functions ? If yes, State it for all trigonometric functions.
Define convergence of sequence and series.
Suppose that $z_n = x_n + iy_n$ $(n = 1, 2, 3 \ldots)$ and $S = X + iY$ then prove that $\sum_{n=1}^{\infty} z_n = S$ iff $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$.

12 Suppose that $f(z) = u(x,\ y) + iv(x,\ y), z_0 = x_0 + iy_0$, and $w_0 = u_0 + iv_0$. Then prove that $\lim\limits_{z \to z_0} f(z) = w_0$ if and only if $\lim\limits_{(x,y) \to (x_0,y_0)} u(x,y) = u_0$ and $\lim\limits_{(x,y) \to (x_0,y_0)} v(x,y) = v_0$.

13 Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differential except at the origin.

14 Derive Cauchy-Riemann equations.

OR

Prove that the necessary conditions for a function $f(z) = u(x,y) + i\,v(x,y)$ is analytic at all points of a region R are (i) $u_x = v_y$ and (ii) $u_y = -v_x$.

15 Verify that $f(z) = e^{-y}\sin x - ie^{-y}\cos x$ is entire.

16 Show that the function $w = \sin z$ satisfies the Cauchy-Riemann equations. Find its derivative.

17 Prove that Cauchy – Riemann conditions are not sufficient for differentiability.

OR

The function f is defined as $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}; & z \neq 0 \\ 0 & z = 0 \end{cases}$, then show that $f(z)$ is not analytic at $z = 0$; even if it satisfies Cauchy-Riemann equations at the origin.

18 Define harmonic function. Find the Harmonic conjugate of $y^3 - 3x^2y$ and corresponding analytic function in terms of z.

19 If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z.

20 If $f(z) = u(r,\ \theta) + i\,v(r,\ \theta)$ be an analytic function and $u = -r^3 \sin 3\theta$ then find a function $\mathrm{v}(r,\ \theta)$ and also express the function $f(z)$ in terms of z.

21 Derive Laplace equation in Polar form.

22 Define: Harmonic conjugate of a function, Entire function.

If $f(z) = u(x,y) + i\,v(x,y)$ is analytic in domain D with non-zero constant modulus, then prove that the function f is constant.

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