Assignment-1

KKSJ MANINAGAR SCIENCE COLLEGE

B. Sc. (Sem-V)

MAT-302 (Analysis – I)

Date: 31-07-2017

1 If $f: A \to B$ and G, H are subsets of B then prove the following.
(i) $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$
(ii) $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$
(iii) $G \subseteq H \Rightarrow f^{-1}(G) \subseteq f^{-1}(H)$.

2 Prove that an upper bound "α" of a non empty set S is the supremum of S if and only if for every $\epsilon > 0$, there exists an element s_ϵ in S such that $\alpha - \epsilon < s_\epsilon$.

3 State and prove Archimedean Property. Using this prove that if $S = \left\{\frac{1}{n}, n \in N\right\}$, then $\inf S = 0$.

4 State and prove rational density theorem.

OR

For every pair of real numbers a and b with $a < b$, prove that there is a rational number r such that $a < r < b$.

5 Prove that there is a real number x such that $x^2 = 2$.

6 Prove that $\sqrt{2}$ is not rational.

7 Prove that the set of all rational numbers is countable.

8 If $I_n = [a_n, b_n], n \in N$ is a nested sequence of closed and bounded intervals then prove that there exists a number $\xi \in R$ such that $\xi \in I_n$ for all $n \in N$.

9 Show by definition that the sequence $\{x_n\}$ defined by $x_n = \frac{3n+4}{2n}$ is convergent. For $\epsilon = 0.001$ find the smallest positive integer n which satisfies the condition of definition.

10 For $0 < |c| < 1$, prove that $c^n \to 0$.

11 Discuss the convergence of $\{(-1)^n\}$ using definition.

12 Prove by definition that $\lim_{n \to \infty} \frac{\cos n + 3 \sin n}{n^2 + n - 60} = 0$.

13 Suppose that $\lim_{n \to \infty} x_n = a$ and $\lim_{n \to \infty} y_n = b$. Then prove that the sequences $\{x_n \pm y_n\}$ are convergent, and $\lim_{n \to \infty} (x_n \pm y_n) = a \pm b$.

14 Suppose that $\lim_{n \to \infty} x_n = a$ and $\lim_{n \to \infty} y_n = b$. Then prove that the sequence $\{x_n y_n\}$ is convergent, and $\lim_{n \to \infty} x_n y_n = ab$.

15 Suppose that $\{x_n\}$ and $\{y_n\}$ are sequences such that $x_n \to a, y_n \to b$, and $x_n \le y_n$ for all n. Then prove that $a \le b$.

16 State and prove Sandwich theorem. Show that $\lim_{n \to \infty} \frac{1}{n^p} = 0$, for any integer $p \ge 2$.

17 Prove that a bounded above increasing sequence is convergent.

18 Suppose $0 < c < 1$, then prove that $\{nc^n\}$ converges to 0.

19 Discuss the convergence of the sequence $\{x_n\}$, $x_1 = 1$ and for $n \geq 1$, $x_{n+1} = \frac{2x_n+3}{4}$. Find limit, if convergent.

20 If $\{x_n\}$ converges to the value a, the prove that every subsequence of this sequence converges to a.

21 State and prove nested interval theorem.

OR

Suppose $\{I_n\}, I_n = [a_n,\ b_n]$, is a sequence of closed intervals such that, for all $n, I_{n+1} \subseteq I_n$. Then prove that

(i) The sequence $\{a_n\}$ and $\{b_n\}$ converge, and $\bigcap_{n=1}^{\infty} I_n = [\alpha,\ \beta]$, where $\alpha = \lim_{n\to\infty} a_n$ and $\beta = \lim_{n\to\infty} b_n$.

(ii) Moreover, if $\lim_{n\to\infty} l(I_n) = \lim_{n\to\infty}(a_n - b_n) = 0$, then $\alpha = \beta$ is the point common to the interval I_n.

22 State and prove Bolzano-Weierstrass Theorem.

OR

Prove that every bounded sequence contains a convergent subsequence.

23 Prove that a real sequence is Cauchy iff it is convergent.

24 If $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$, then prove that $2 < \lim_{n\to\infty} S_n < 3$.

25 Prove that the sequence $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$ converges.

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