K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-I

T. Y. B. Sc. (Sem-V)

MATHEMATICS

MAT-301 (Linear Algebra-II)

Q-1 Define a composition of linear Mappings and show that the composition of linear mappings is a linear mapping.

Q-2 Define addition of two linear Mappings and show that the addition of linear mappings is a linear mapping.

Q-3 Define a scalar multiplication of a linear Mapping show that a scalar multiplication of a linear mapping is a linear mapping.

Q-4 Prove in usual notations that L(U,V) is a vector space.

Q-5 Define : (i) Homogeneous Operator equation

(ii) Non-homogeneous Operator equation.

(iii) A Linear Variety.

Q-6 If $T : U \to V$ is linear map , $v_0 \notin R(T)$ then prove that the operator equation $T(u) = v_0$ has no solution.

Q-7 If $T : U \to V$ is linear map , $v_0 \in R(T)$ and if $T(u) = \bar{0}_V$ has only trivial solution $u = \bar{0}_U$ then prove that the operator equation $T(u) = v_0$ has a unique solution.

Q-8 If $T : U \to V$ is linear map , $v_0 \in R(T)$ and if $T(u) = \bar{0}_V$ has a non-trivial solution $u_0 \neq \bar{0}_U$ then prove that $T(u) = v_0$ has an infinite solution set namely $u_0 + N(T)$.

Q-9 If the linear map $T : V_3 \to V_3$ is defined as $T(e_1) = e_1 - e_2$, $T(e_2) = e_2 - e_3$, $T(e_3) = e_3 - e_1$ then solve the operator equation $T(x_1, x_2, x_3) = (5, 3, 1)$.

Q-10 If $T:V_3 \to V_3$ is the linear map defined as $T(x,y,z)= (x+ y,y+ z,z+x)$, $\forall (x,y,z) \in V_3$ then solve the operator equation $T(x,y,z) = (2,4,6)$.

Q-11 Solve the operator equation $T(x) = (1,2,3)$ if the linear map $T : V_2 \to V_3$ is defined as $T(x) = T(x_1, x_2) = (x_1 , x_1 + x_2 , x_2)$, $\forall x = (x_1, x_2) \in V_2$

Q-12 If the linear map $T : V_3 \to V_2$ is defined as

$T(x) = T(x_1, x_2 , x_3) = (x_1 - x_2 , x_1 + x_3)$, $\forall x = (x_1, x_2 , x_3) \in V_3$

then solve the operator equation $T(x) = (1, 4)$.

Q-13 Define a linear functional and give one example of it.

Q-14 Show that the function $f : R^3 \rightarrow R$ defined as
$f(x) = f(x_1, x_2, x_3) = x_1 - x_2 + x_3, \; \forall \; x = (x_1, x_2, x_3) \in R^3$
is a linear functional on R^3.

Q-15 Show that the trace function $tra : \mu_{n,n} \rightarrow R$ defined as
$tra A = \sum_{i=1}^{n} a_{i,i}$, for matrix $A=(a_{ij})$ of order $n \times n$ is a linear functional.

Q-16 Show that the integral function $\mathscr{I}: \mathfrak{C}[a, b] \rightarrow R$ defined as
$\mathscr{I}(f) = \int_a^b f(x)dx, \; \forall \; f \in \mathfrak{C}[a, b]$ is a linear functional on $\mathfrak{C}[a, b]$.

Q-17 Show that the i[th] co-ordinate function $f^{\,i} : R^n \rightarrow R$ defined as
$f^{\,i}(x) = f^{\,i}(x) = f^{\,i}(x_1, x_2, \ldots, x_i, \ldots, x_n) = x_i, \; \forall x = (x_1, x_2, \ldots, x_i, \ldots, x_n) \in R^n$
is a linear functional on R^n.

Q-18 State and prove the dual basis existence theorem.

Q-19 Find the dual basis of the basis B = { (2,3), (1,4)} of R^2.

Q-20 Find the dual basis of the basis B = {(1,0,0), (1,1,0), (1,1,1)} for the vector space V_3.

Q-21 Find the dual basis of the basis B = {(–1,1,1), (1, –1,1), (1,1, –1)} for the vector space V_3.

Q-22 Define a bilinear form. If for $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3) \in R^3$, the map
$f : R^3 \times R^3 \rightarrow R$ is defined as $f(x, y) = x_1 y_2 - 3x_2 y_3 + x_3 y_1$ then show that
f is a bilinear form on R^3.

Q-23 Define a bilinear form. Show that the map $\emptyset : V_2 \times V_2 \rightarrow R$ defined as
$\boldsymbol{\emptyset(x, y) = x_1 y_1 - x_1 y_2 + 2x_2 y_1 + 3x_2 y_2}$, $\forall$ for $x = (x_1, x_2)$, $y = (y_1\, y_2) \in V_2$
is a bilinear form.

Q-24 Define an inner product and give one example of it.

Q-25 If for $x = (x_1, x_2)$, $y = (y_1\, y_2) \in V_2$, the map $< , > : V_2 \times V_2 \rightarrow R$ is defined as

$<x , y> = x_1 y_1 - x_1 y_2 - x_2 y_1 + 2x_2 y_2$

then show that $< , >$ is an inner product on V_2.

Q-26 If for $x = (x_1, x_2)$, $y = (y_1\, y_2) \in R^2$, the map $< , > : R^2 \times R^2 \rightarrow R$ is defined as

$<x , y> = x_1 y_1 + x_1 y_2 + x_2 y_1 + 5x_2 y_2$

then show that $< , >$ is an inner product on R^2.

Q-27 If for $x = (x_1, x_2)$, $y = (y_1\, y_2) \in R^2$ the map $< , >$ is defined as

$<x , y> = x_1 [y_1 - y_2] + x_2 [2 y_2 - y_1]$ then show that $< , >$ is an inner product on R^2.

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