K. K. SHAH JARODWALA MANINAGAR SCIENCE COLLEGE, Ahmedabad.

Assignment-I

S. Y. B. Sc. (Sem-III)

MATHEMATICS

MAT-202 (Linear Algebra-I)

Q-1 Define a relation, Reflexive relation, Symmetric relation, transitive relation and an equivalence relation.

Q-2 Define a Binary Operation, an associative Binary Operation and a commutative binary operation.

Q-3 Define a real vector space and a complex vector space.

Q-4 Justify whether the set Z of all integers form a vector space under usual addition and scalar multiplication ?

Q-5 If $\mathscr{F}$ (I) represent the set of all real valued functions defined on an interval I then $\mathscr{F}$ (I) forms a vector space under usual addition and scalar multiplication of mappings. As a part of this Show that ($\mathscr{F}$ (I) , +) is a commutative group.

Q-6 If V is a real Vector Space then Prove that V has a unique additive identity.

Q-7 If V is a real Vector Space then Prove that every element of V has a unique additive inverse in V.

Q-8 If V is a real Vector Space then Prove in usual notations that

$0 \cdot u = \bar{0}$, for every element $u \in$ V.

Q-9 If V is a real Vector Space then Prove in usual notations that

$\alpha \cdot \bar{0} = \bar{0}$ for every scalar α

Q-10 If V is a real Vector Space then Prove in usual notations that

$(-1) \cdot u = -u$, for every $u \in$ V.

Q-11 Define a Vector Subspace (Or A Subspace) of a real vector space V.

Q-12 State and prove the necessary and sufficient condition for a subset S of a vector Space V to be its Subspace.

Q-13 Prove that $U \cap W$ is a subspace of V for any two subspaces U and W of a vector space V.

Q-14 If U and W are subspaces of a vector space V then Prove that $U + W$ also is a subspace of V.

Q-15 Show that S = $\{(x,y,z) \in V_3 / x+y-z = 0\}$ is a subspace of the vector space V_3.

Q-16 Show that S = $\{ (x,y,z) / x-y-z=0 \}$ is a subspace of the vector space V_3. Also find *dim*S.

Q-17 Determine whether U = $\{ (x_1+ x_2, x_1- x_2, 0) / x_1, x_2 \in R \}$ is a subspace of V_3 or not. .

Q-18 Determine whether U = $\{ (x_1, x_2, 3) / x_1, x_2 \in R \}$ is a subspace of V_3 or not. .

Q-19 Show that S = $\{ (x,y,z) / z = x-y \}$ is a subspace of the vector space V_3.

Q-20 Show that S = $\{ (x,y,z) / x+ y-z = 0 \}$ is a subspace of the vector space V_3.

Q-21 Define a Linear combination of vectors and the Span of a set.

Q-22 Define linear span of a set and prove in usual notations that [S] is the smallest subspace of a vector space V containing S for every nonempty subset S of V.

Q-23 Prove in usual notations that

[(1,1,0) ,(0,0,1) , (5,5,5)] = [(3,3,0) ,(0,0,3)].

Q-24 Determine whether the following statements are true or false :

(i) The set Z of all integers is a vector space under usual addition and scalar multiplication.

(ii) Every vector space has a proper subspace.

(iii) The set N of natural numbers is a vector space under usual addition and scalar multiplication.

(iv) Every subspace is a vector space structure.

(v) If V_1 is a subspace of V_2 then $V_1 \subset V_2$.

(vi) If V_1 is a vector space and $V_1 \subset V_2$ then V_2 also must be a vector space.

(vii) Every vector space has a sub space.

(viii) The zero space $V_0 = \{\bar{0}\}$ has no subspace .

(ix) R^+ is a subspace of R.

Q-25 Answer the followings in short :

(i) What is a Scalar ?

(ii) What is a vector ?

(iii) Define a Field.

(iv) Define a finite dimensional vector space.

(*v*) Define the Dimension of a finite dimensional vector space.

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