Assignment-1

KKSJ MANINAGAR SCIENCE COLLEGE

SEMESTER : III

MAT – 201 (Advanced Calculus-I)

Date: 31-07-2017

1 Find the limit using definition $\lim_{(x,\ y) \to (1,\ 1)} \frac{x^2+y^2}{x+y}$, if exists.

2 Using definition evaluate $\lim_{(x,\ y) \to (1,\ 2)} (xy - 3x + 4)$.

3 Define limit of function of two variables. Use this definition to find $\lim_{(x,\ y) \to (2,\ 1)} \frac{2x+y}{3y-x}$.

4 Define iterated limits and find the iterated limits of the following functions

(i) $f(x,\ y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & ;\ x^2+y^2 \neq 0 \\ 0 & ;\ x^2+y^2 = 0 \end{cases}$ at $(0,\ 0)$

(ii) $f(x,\ y) = \begin{cases} \frac{\sin(x+y)}{x+y} & ;\ x+y \neq 0 \\ 1 & ;\ x+y = 0 \end{cases}$ at $(0,\ 0)$.

5 Let function $\phi(x)$ be continuous at a point $(a,\ \phi(a)) = (a,\ b)$ and $\lim_{(x,\ y) \to (a,\ b)} f(x,\ y)$ exists and is equal to $l \in R$, then prove that $\lim_{x \to a} f(x,\ \phi(x))$ exists and is equal to l.

6 Evaluate: $\lim_{(x,\ y) \to (0,\ 0)} \frac{\sin(x+y)}{x+y}$

7 Evaluate the iterated limits of $f(x,\ y) = \frac{x^2-y^2}{x^2+y^2}$ as $(x,\ y) \to (0,\ 0)$.

8 Discuss the continuity of $f(x,\ y) = \begin{cases} x^2 \sin\frac{y}{x} & ;\ x \neq 0 \\ 1 & ;\ x = 0 \end{cases}$ at $(0,\ 0)$.

9 Discuss the continuity of $f(x,\ y) = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & ;\ x \neq 0 \\ 0 & ;\ x = 0 \end{cases}$ at point $(0,\ 0)$.

10 Find the limit $\lim_{(x,\ y) \to (0,\ 0)} \frac{x^5 y}{x^{10}+y^2}$, if exists.

11 State and prove Young's theorem.

12 State and prove Schwarz's theorem.

13 If function $z = f(x,\ y)$, defined on an open set $E \subset R^2$, is differentiable at point $(x,\ y) \in E$, then prove that its partial derivatives f_x and f_y exist at point $(x,\ y)$. Does the converse hold ? Justify.

14 If $(a,\ b)$ be a point of the domain of the definition of the function f such that (i) f_x is continuous at $(a,\ b)$ and (ii) f_y exists at $(a,\ b)$, then prove that f is differentiable at $(a,\ b)$.

15 Let $f(x,\ y)=\begin{cases}\frac{x^3-y^3}{x^2+y^2} & ;\ x^2+y^2\neq 0\\ 0 & ;\ x^2+y^2=0\end{cases}$. Show that $f_x(0,\ 0), f_y(0,\ 0)$ exist but f is not differentiable at $(0,\ 0)$.

16 Find the directional derivative of function $f(x,\ y)=\begin{cases}\frac{xy^2}{x^2+y^4} & x\neq 0, y\neq 0\\ 0 & x=0, y=0\end{cases}$ at point $(0,\ 0)$ in the direction of $\left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$.

17 If $u=\log(x^2+y^2+z^2)$, then prove that $x\frac{\partial^2 u}{\partial y\,\partial z}=y\frac{\partial^2 u}{\partial z\,\partial x}=z\frac{\partial^2 u}{\partial x\,\partial y}$.

18 If $u=\log(x^2+y^2+z^2)$, then prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{2}{x^2+y^2+z^2}$.

19 If Z = f(x,y) is a real function defined on a non empty set E of R^2 and if $f_x(x,y)$ & $f_y(x,y)$ exist and are continuous at $(x,y)\in E$ then prove that the function f is differentiable at point $(x,y)\in E$.

20 If $F(x,y,u,v)=x^2+y^2+u^2+v^2-1=0$ and $G(x,y,u,v)=x^2+2y^2-u^2+v^2-1=0$, then prove that $\frac{\partial^2 u}{\partial x^2}=-\frac{9u^2}{x^3}-\frac{3}{x}$.

21 If $u=f(r)$, $r^2=x^2+y^2+z^2$, then prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$.

22 If $u=\log(x^2+y^2)$, then prove that u is harmonic function of x and y.

23 Find f_{xx}, f_{xy} and f_{yy} for the function $f(x,\ y)=\begin{cases}\sin^{-1}\left(\frac{x}{y}\right) & ;\ y\neq 0\\ 0 & ;\ y=0\end{cases}$.

24 If $f(x,\ y)=\begin{cases}\frac{xy\,(x^2-y^2)}{x^2+y^2} & ;\ (x,\ y)\neq(0,\ 0)\\ 0 & ;\ (x,\ y)=(0,\ 0)\end{cases}$ then find $f_{xx}(0,\ 0), f_{xy}(0,\ 0), f_{yx}(0,\ 0)$ and $f_{yy}(0,\ 0)$.

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